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# The Beam-Beam Interaction in Lattices with Non-Identical Insertions

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## 1. INTRODUCTION

If one starts out with a lattice with 6 identical insertions, then the beam-beam interaction has the periodicity 6, and the stop bands, due to the non-linear beam-beam forces, are at the resonance lines $n\upsilon_x + m\upsilon_y = q$ where q is a multiple of 6. If now some of these insertions are replaced with insertions having different $\beta_x, \beta_y$ at the interaction point, then the beam-beam interaction effects could be worsened because the periodicity of the beam-beam interaction may be lowered. This paper shows that if the different insertions introduced obey certain conditions listed below, then the beam-beam stop-bands and the beam-beam $\upsilon$-shift are not changed by the introduction of the different insertions. These conditions on the insertions are the following:

1) $\beta_y/\beta_x$ is the same for all insertions.
2) $\Delta\psi_x, \Delta\psi_y$ the change in the betatron phase between crossing points is not changed by the different insertions.
3) $\beta_x, \beta_y$ and the $\sigma_x, \sigma_y$ of the beam may be considered to be constant along the path where the beams interact.
4) $\alpha/\beta_x^{1/2}$ is the same for all insertions, where $\alpha$ is the crossing angle of the beams.

Usually, the above conditions are easy to fulfill, as they appear to occur naturally. It would seem desirable to require that, if possible, all insertions introduced into the lattice should obey the above 4 conditions.

## 2. Derivation of the Conditions

The beam-beam stop bands are given by;

$$\Delta\upsilon_{nm} \sim \int ds\, \beta_x^{n/2}\, \beta_y^{n/2}\, D_x^n\, D_y^m\, V \exp\left[i\,(n\psi_x + m\psi_y)\right], \tag{2.1}$$

where we have dropped factors which are the same at all the insertions in the lattice $D_x = \partial/\partial x$ and $D_y = \partial/\partial y$. $V = V(x,y)$ is the potential function that gives the field due to the beam.

The factor $\beta_x^{n/2}\ \beta_y^{n/2}$ in Eq. (2.1) indicates that insertions with larger $\beta_x,\beta_y$ contribute more to the stop bands. However at larger $\beta_x,\beta_y$ the beam becomes larger and $D_x^n\ D_y^m\ V$ becomes smaller by the same factor $1/(\beta_x^{n/1}\ \beta_y^{m/2})$ provided $\beta_y/\beta_x$ and $\alpha/\beta_x^{1/2}$ are the same for all insertions.

This can be verified by starting with the expression for V(x,y)

$$V = -\frac{\lambda}{4\pi\varepsilon_0}\int_0^{\infty} dt\ \frac{1 - \exp\left[-\left(x^2/(2\sigma_x^2 + t) + y^2/(2\sigma_y^2 + t)\right)\right]}{(2\sigma_x^2 + t)^{1/2}\ (2\sigma_y^2 + t)^{1/2}}\ , \tag{2.2}$$

and differentiating under the integral sign to find $D_x^n, D_y^m\ V$ and evaluating the result at $x = \alpha s$, $y = 0$. This involves some manipulation of the integrand.

The condition (2) on $\Delta\psi_x$ and $\Delta\psi_y$ between crossing points guarantees that the contributions due to the 6 interaction regions will add up with the same phases.

The $\upsilon$-shifts are given by Eq. (2.1) without the phase factor and by putting $n = 2$, $m = 0$ for $\Delta\upsilon_x$ and $n = 0$, $m = 2$ for $\Delta\upsilon_y$. Thus the above arguments for the stop bands work for the $\upsilon$-shifts above.